Denseness of $\mathbb{Q}$: Given any $a<b$, $a,b \in \mathbb{R}$, there exists an $r \in \mathbb{Q}$ s.t. $a<r<b$.

Proof: Need to find $m,n \in \mathbb{Z}$ s.t. $a<\frac{m}{n}<b$. assume that $n>0 \iff na<m<nb$. First, determine n: We have that $b-a>0$, then we can use the Archimedean property for $b-a$ and 1 to conclude that there exists an $n \in \mathbb{N}$ s.t. $nb-na>1$. Next we will look for $m \in \mathbb{Z}$ s.t. $m \in (na, nb)$: (a) Consider the $\max\{|an|,|bn|\}$ and $1 \overset{A.P.}{\Longrightarrow} \exists k \in \mathbb{N}$ s.t. $k \cdot 1 > \max\{|an|,|bn|\}$ $\Rightarrow |an|<k, |bn|<k \Rightarrow \begin{matrix} -k<an<k \\ -k<bn<k \end{matrix} \Rightarrow$ $-k<an<bn<k$. (b) Define $S=\{j \in \mathbb{Z} : -k \le j \le k, j>an\}$ - notice that S is a set of finitely many elements $\Rightarrow \min S$ exists; denote $m=\min S$. By definition of S: $m>an \Rightarrow$ need to show that $m<bn$. We have that $m-1 \notin S \Rightarrow m-1<an \Rightarrow m<an+1$, but $an+1<bn \therefore m<bn \Rightarrow an<m<bn$ $\boxtimes$