Def: A sequence is an ordered set of ∞ many real #'s.

Ex: (a) $\{1, 2, 3, 4, 5, 6, \ldots\}$ - is a sequence

(b) $\{2, 1, 3, 4, 5, 6, \ldots\}$ - is a sequence, but it is a different sequence than in (a).

(c) $\{2, 1, 4, 3, 6, 5, 8, 7, \ldots\}$ - is a sequence, different from (a) or (b).

(a), (b), (c) are sequences of natural #'s.

(d) $\{1, -1, 1, -1, 1, -1, \ldots\}$ - sequence;

(e) $\{1, \frac{1}{2}, \frac{1}{4}, \ldots\}$ - sequence;

alternatively: (a) $s_n = n$, $n \geq 1$ (index; s_n: n-th element of the sequence)

or $\{n \mid n \in \mathbb{N}\}$ or $\{n\}_{n\in\mathbb{N}}$ or $\{n\}_{n=1}^{N}$

(b) $s_1 = 2$, $s_2 = 1$, $s_n = n$, $n \geq 3$

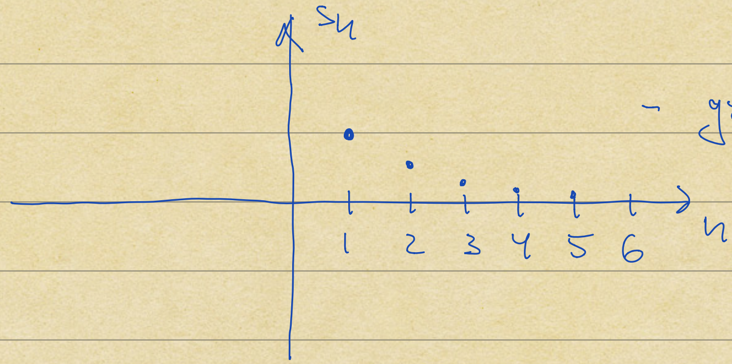
(c) $s_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even} \end{cases}$, $n \in \mathbb{N}$

(d) $s_n = (-1)^{n+1}$, $n \in \mathbb{N}$; $\{(-1)^{n+1}\}_{n=1}^{\infty}$

(e) $s_n = \frac{1}{2^{n-1}}$, $n \in \mathbb{N}$; $\{\frac{1}{2^n}\}_{n=1}^{\infty}$

Can specify a sequence by stating how s_n depends on n $\Rightarrow$ can think of a sequence $\{s_n\}$ as a function $s(n)$, e.g., in the case (e) above $s(n) = \frac{1}{2^n}$, $n \in \mathbb{N}$ $\Rightarrow$ $s(n)$ is a function from $\mathbb{N}$ to $\mathbb{R}$.

$\Rightarrow$ Can graph a sequence: Say $s_n = \frac{1}{2^{n-1}}$, $n \in \mathbb{N}$



- graph is a set of ∞ many discrete points.

(f) Sequences can also be defined recursively:

$s_1 = 1$, $s_{n+1} = 2s_n + 3$ for each $n \geq 1$, then

$\{1, 5, 13, 29, \ldots\}$

(g) $s_1 = 2$, $s_2 = 4$, $s_n = s_{n-1} + s_{n-2}$ if $n \geq 3$

$\{2, 4, 6, 10, 16, 26, \ldots\}$

(h) $s_n = \frac{n+3}{n+1}$, $n \geq 0$ $\Rightarrow$ $s_0 = \frac{0+3}{0+1} = 3$, $s_1 = \frac{1+3}{1+1} = \frac{4}{2} = 2$

$s_2 = \frac{2+3}{2+1} = \frac{5}{3}$, $s_3 = \frac{3+3}{3+1} = \frac{3}{2}, \ldots$

$\Rightarrow \{3, 2, 5/3, 3/2, \ldots\}$

(i) Consider the sequence $s_n = \frac{1}{n}$, $n \geq 1 \Rightarrow$

$\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$ $\therefore$ as n tends to ∞,

s_n tends to 0. We would like to say that

$\lim_{n\to\infty} s_n = 0$. Likewise, in (h), $\lim_{n\to\infty} \frac{n+3}{n+1} = 1$

To make the notion of limit precise, what is meant by the limit? Roughly speaking:

$$\lim_{n\to\infty} s_n = L \quad \text{if} \quad s_n \text{ can be made}$$

arbitrarily close to L by taking n sufficiently large.

How do we make precise the statement

(1) "s_n can be made arbitrarily close to L"?

$\Leftrightarrow$ "given any number $\varepsilon > 0$, can make the distance between s_n and L smaller than ε"

$\Leftrightarrow$ "given any number $\varepsilon > 0$, $|s_n - L| < \varepsilon$"

(2) "n is sufficiently large"? $\Leftrightarrow$ "there exists

$N \in \mathbb{R}$ s.t. if $n > N$ ..."

$\Rightarrow$ <u>Def of the limit</u>: The limit of s_n as n goes to infinity is equal L,

$$\lim_{n\to\infty} s_n = L \quad \text{if}$$

for any $\varepsilon > 0$, there exists a $N \in \mathbb{R}$ s.t. $|s_n - L| < \varepsilon$ if $n > N$.

In a compact form: $\lim_{n\to\infty} s_n = L$ if $\forall \varepsilon > 0$, $\exists N \in \mathbb{R}$ s.t. $n > N \Rightarrow |s_n - L| < \varepsilon$.