

Pb2: $\forall \varepsilon > 0$, set $N=1 \Rightarrow \forall n > N$ we have
that $|s_n - s| = |a - a| = 0 < \varepsilon \Rightarrow$

$$\lim_{n \rightarrow \infty} s_n = a$$

Pb3:

Suppose that $\lim_{n \rightarrow \infty} s_n = L_1$, $\lim_{n \rightarrow \infty} s_n = L_2$, where $L_1 \neq L_2$

Choose any
 $\varepsilon > 0$:

$$\begin{array}{ccc} \Downarrow & & \Downarrow \\ \exists N_1 \in \mathbb{R} \text{ s.t.} & & \exists N_2 \in \mathbb{R} \text{ s.t.} \\ n > N_1 \Rightarrow & & n > N_2 \Rightarrow \\ \underbrace{|s_n - L_1| < \varepsilon} & & \underbrace{|s_n - L_2| < \varepsilon} \end{array}$$

If $n > N = \max(N_1, N_2) \Rightarrow |s_n - L_1| < \varepsilon, |s_n - L_2| < \varepsilon$

$$\begin{aligned} \Rightarrow |L_2 - L_1| &= |L_2 - s_n + s_n - L_1| \stackrel{\text{T.I.}}{\leq} |s_n - L_2| + |s_n - L_1| \\ &< \varepsilon + \varepsilon = 2\varepsilon \end{aligned}$$

$$\Rightarrow |L_2 - L_1| < 2\varepsilon \text{ for any } \varepsilon > 0 \Rightarrow |L_2 - L_1| = 0$$

$\Rightarrow L_2 = L_1$ - contradiction $\Rightarrow$ The limit is
unique.

Pb4: See book.

Pl 5: Suppose that the statement of the problem is not true and $\exists \epsilon > 0$ s.t.
 $\inf E + \epsilon \leq a$ for $\forall a \in E \Rightarrow \inf E + \epsilon$ is a lower bound on E - contradicts the definition of E as the greatest lower bound $\square$