Problem 8. Consider a large tank holding 1000 L of water into which a brine solution of salt begins to flow at a rate of 6 L/min. The solution inside the tank is kept well stirred and is flowing out of the tank at a rate of 6 L/min. If the concentration of salt in the brine entering the tank is 1 kg/L, determine the concentration of salt in the tank after one hour.

If $m(t)$ is the mass of salt in the tank at time $t \Rightarrow C(t)$ is the concentration of salt in the tank at the same time, where $C(t) = \frac{m(t)}{1000}$. The rate of change of m is

$$m'(t) = 6 - \frac{6m}{1000}; \quad m(0) = 0$$

this is a linear IVP: $m' + \frac{3}{500} m = 6$, $\mu(t) = e^{\int \frac{3}{500} dt} = e^{3t/500}$, then

$\left(e^{3t/500} m\right)' = 6e^{3t/500}$; Integrating, $e^{3t/500} m = 1000 e^{3t/500} + C \Rightarrow m(t) = 1000 + Ce^{-3t/500}$. Use the initial condition:

$0 = m(0) = 1000 + C \Rightarrow C = -1000$ and $m(t) = 1000\left(1 - e^{-3t/500}\right)$, hence $m(1) = 1000\left(1 - e^{-3/500}\right)$.

Problem 9. Suppose that the temperature of the cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 200 degrees when freshly poured, and one minute later has cooled to 190 degrees in a room at 70 degrees, determine when the coffee reaches a temperature of 150 degrees.

Solution: $T(0) = 200$ and $T_z = 70 \Rightarrow T(t) = 70 + (200-70)e^{-kt} = 70 + 130e^{-kt}$; $190 = T(1) = 70 + 130e^{-k} \Rightarrow$
$120 = 130e^{-k}$ and $e^{-k} = \frac{12}{13} \Rightarrow T(t) = 70 + 130\left(\frac{12}{13}\right)^t$. Need to solve for t: $150 = 70 + 130\left(\frac{12}{13}\right)^t \Rightarrow \left(\frac{12}{13}\right)^t = \frac{8}{13}$;
$\Rightarrow t \ln\frac{12}{13} = \ln\frac{8}{13} \Rightarrow t = \frac{\ln 8/13}{\ln 12/13}$.