**The University of Akron**
**Department of Mathematics**
**Differential Equations**
**Spring 2023, Golovaty**

## REVIEW PROBLEMS FOR TEST 1

**Problem 1**. Verify whether the family $y = C_1 e^{2x} + C_2 e^{-x}$ of functions, where $C_1$ and $C_2$ are arbitrary constants is a two-parameter family of solutions of the ODE $y'' - y' - 2y = 0$. What is the order of the ODE? Is it linear or nonlinear?

**Problem 2**. For each of the following differential equations state the region in the $xy$-plane where the existence of a unique solution through any specific point is guaranteed by the existence and uniqueness theorem.

**(a)** $y' = \left(x^2 + y^2\right)^{1/2}$, **(b)** $(x + y)y' = x - y$.

**Problem 3**. Solve the given differential equation by separation of variables

**(a)** $y' = \frac{y}{x^2}$, **(b)** $y' + y^2 \sin x = 0$,

**(c)** $y' = 1 + x + y^2 + xy^2$, **(d)** $(1 + y^2)y' = x^2$.

**Problem 4**. Solve the given linear differential equation

**(a)** $x^2 y' + 3xy = \frac{\sin x}{x}, \quad x < 0$, **(b)** $y' + 2y/x = e^x/x, \quad x > 0$,

**(c)** $y' + 3y = x + e^{-2x}$, **(d)** $y' - 2y = x^2 e^{2x}$.

**Problem 5**. Solve the given differential equation by using an appropriate substitution

**(a)** $y' = \frac{y-x}{y+x}$, **(b)** $(y^2 + yx)dx + x^2 dy = 0$,

**(c)** $xy' - y = \sqrt{x^2 + y^2}$, **(d)** $x^2 y' + 2xy - y^3 = 0, \quad x > 0$.

**Problem 6**. Solve the given initial value problem by any method you wish

**(a)** $y' + 2y = xe^{-2x}\,, \quad y(1) = 0\,,$ **(b)** $y' = \frac{2x}{(y+x^2y)}\,, \quad y(0) = -2\,,$

**(c)** $\sin 2x dx + \cos 3y dy = 0\,, \quad y(\pi/2) = \pi/3\,.$

**Problem 7**. Show that if $a$ and $\lambda$ are positive constants, and $b$ is any real number, then every solution of the equation

$$y' + ay = be^{-\lambda x}\,,$$

has the property that $y \to 0$ as $x \to \infty$. *Hint*: Consider the cases $a = \lambda$ and $a \neq \lambda$ separately.

**Problem 8**. Consider a large tank holding 1000 $L$ of water into which a brine solution of salt begins to flow at a rate of 6 $L/min$. The solution inside the tank is kept well stirred and is flowing out of the tank at a rate of 6 $L/min$. If the concentration of salt in the brine entering the tank is 1 $kg/L$, determine the concentration of salt in the tank after one hour.

**Problem 9**. Suppose that the temperature of the cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 200 degrees when freshly poured, and one minute later has cooled to 190 degrees in a room at 70 degrees, determine when the coffee reaches a temperature of 150 degrees.

**Problem 10**. If initially there are 50 grams of a radioactive substance and after 3 days there are only 10 grams remaining, what percentage of the original amount remains after 4 days?

**Problem 11**. Determine whether the given differential equation is exact. If it is exact, solve it.

**(a)** $(2x + 3) + (2y - 2)y' = 0\,,$ **(b)** $(2xy^2 + 2y) + (2x^2y + 2x)y' = 0\,,$

**(c)** $(xlny + xy)dx + (ylnx + xy)dy = 0,\ x > 0,\ y > 0\,,$ **(d)** $\frac{dy}{dx} = -\frac{ax+by}{bx+cy}\,.$

**Problem 12**. Verify whether the functions $y_1(x) = e^{2x}$, $y_2(x) = e^{-x}$, and $y_3(x) = e^x$ form a fundamental family of solutions of the ODE $y''' - 2y'' - y' + 2y = 0$.

**Problem 13**. Verify whether the functions $f_1(x) = x^2$, $f_2(x) = 2x^2 - 3x$, $f_3(x) = x$, and $f_4(x) = 1$ are linearly independent. Do not use Wronskian to solve this problem.

**Problem 14**. Find a second solution of the differential equation $x^2y'' + 2xy' = 0$ by the method of reduction of order given that $y_1(x) = 1$ is a solution of the equation. For what range of $x$ would you expect your solution to be valid?

**Problem 15**. Find a second solution of the given differential equation by the method of reduction of order

**(a)** $y'' - 4y' - 12y = 0\,,\ y_1(x) = e^{6x}$ **(b)** $x^2y'' + 3xy' + y = 0\,,\ x > 0\,,\ y_1(x) = x^{-1}$