The University of Akron
Department of Mathematics
Differential Equations
Spring 2023, Golovaty

REVIEW PROBLEMS FOR THE FINAL EXAM

Problem 1. Use the method of undetermined coefficients to find the general solution of the equation

$$y'' - 3y' - 4y = 8x + 2\sin x + 5e^{-x}.$$

Problem 2. Use the method of variation of parameters to find the general solution of the equation

$$y'' + y = \frac{1}{\cos x}\,, \quad 0 < x < \frac{\pi}{2}\,.$$

Problem 3. Solve the given second order linear differential equation

(a) $y'' - 2y' + 2y = 0\,,$ **(b)** $9y'' - 6y' + y = 0\,,$

(c) $y'' - y = 0\,,$ **(d)** $y''' + y = 0\,.$

Problem 4. Solve the given initial or boundary value problem.

(a) $y'' + 8y' - 9y = 0\,, \quad y(1) = 1\,, \quad y'(1) = 0\,,$ **(b)** $y'' + y = 0\,, \quad y(\pi/3) = 2\,, \quad y'(\pi/3) = -4\,,$

(c) $y'' + y = 0\,, \quad y'(0) = 0\,, \quad y'(\pi/2) = 2\,.$

Problem 5. Solve the equation $y''+2y'+y = x^2+e^{-x}\ln x$ by combining the methods of undetermined coefficients and variation of parameters.

Problem 6. Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ for $f(t) = e^{-t}\sin t$.

Problem 7. Suppose that a vertical spring has one of its ends attached to the ceiling. Suppose further that a mass of 1 kg extends the spring by 9.8 cm after the mass is affixed to the free end of the spring and the system is brought to rest. If the mass is then moved down from the equilibrium position by 1 m and then released, find the position of the mass at any time $t > 0$ (you can assume that there is no friction in this spring/mass system). Also, find the amplitude, period and frequency of oscillations.

Problem 8. A 0.5 kg mass stretches a spring .098 m. The mass starts from equilibrium with the upward velocity of 8 m/s and the subsequent motion takes place in a medium that offers a damping force numerically equal to 6 times the instantaneous velocity. Assuming that the gravitational acceleration is 9.8 m/s^2, find the position of the mass at any time $t > 0$.

Problem 9. Find the Laplace transform of a given function

$$\textbf{(a).}\ f(t) = 5 - e^{2t} + 6t^2, \quad \textbf{(b).}\ g(t) = t^4 e^{5t} - e^t \cos \sqrt{7} t,$$
$$\textbf{(c).}\ p(t) = (t+1)^2, \quad \textbf{(d.)}\ h(t) = \sin(t + 2\pi).$$

Problem 10. Find the inverse Laplace transform of

$$\textbf{(a).}\ F(s) = \frac{6}{(s-1)^4}, \quad \textbf{(b).}\ F(s) = \frac{s+1}{s^2+2s+10},$$
$$\textbf{(c).}\ F(s) = \frac{6s^2 - 13s + 2}{s(s-1)(s-6)}, \quad \textbf{(d.)}\ F(s) = \frac{s+1}{s^2+4}.$$

Problem 11. Use Laplace transform to find the solution of an initial value problem

$$\textbf{(a).}\ y'' - y' - 2y = 0,\ y(0) = -2,\ y'(0) = 5,$$
$$\textbf{(b).}\ y'' + 4y = 4t^2 - 4t + 10,\ y(0) = 0,\ y'(0) = 3.$$

Problem 12. Find the Laplace transform of a given function

$$\textbf{(a).}\ y(t) = t^2\,\mathcal{U}(t-2), \quad \textbf{(b).}\ y(t) = \left(\sin t + e^{3t}\right) \mathcal{U}(t - 2\pi).$$

Problem 13. Solve the given differential equation

$$\textbf{(a).}\ x^2y'' - 7xy' + 41y = 0,\ x > 0, \quad \textbf{(b).}\ t^2x'' + 3tx' - 4x = 0,\ t > 0,$$
$$\textbf{(c).}\ x^2y'' - 5xy' + 10y = 0,\ x > 0, \quad \textbf{(d).}\ x^2y'' + 3xy' + y = 0,\ x > 0.$$

Problem 14. Solve the initial value problem

$$x^2y'' + xy' + y = 0,\ y(1) = 1,\ y'(1) = 2.$$

Problem 15. Solve the system of linear differential equations by any method you wish

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix} \mathbf{x}.$$