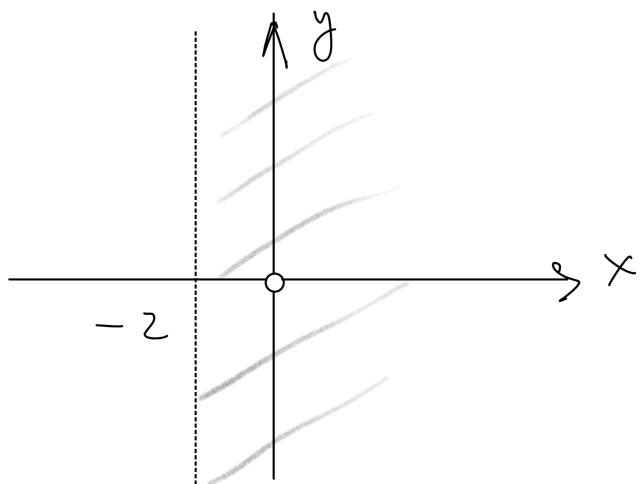
Problem 11. For the function $f(x,y)=\frac{xy\ln(x+2)}{x^2+y^2}$

- Sketch the domain of f.

- Find $\lim_{(x,y)\to(0,0)} f(x,y)$ if it exists.

Domain: $x^2+y^2\neq 0 \Rightarrow (x,y)\neq(0,0)$

$x+2>0 \Rightarrow x>-2$



$$\lim_{(x,y)\to(0,0)} \frac{xy\ln(x+2)}{x^2+y^2}$$

Check the limit along $y=kx$:

$$\lim_{x\to 0} \frac{x\cdot kx\ln(x+2)}{x^2+k^2x^2} = \lim_{x\to 0} \frac{k\cancel{x^2}\ln(x+2)}{(1+k^2)\cancel{x^2}}$$

$$=\frac{k}{1+k^2}\lim_{x\to 0}\ln(x+2) = \frac{k\ln 2}{1+k^2}$$ – this depends

on k $\Rightarrow$ the original limit does not exist.

Problem 12. Sketch several level curves (surfaces) of the function

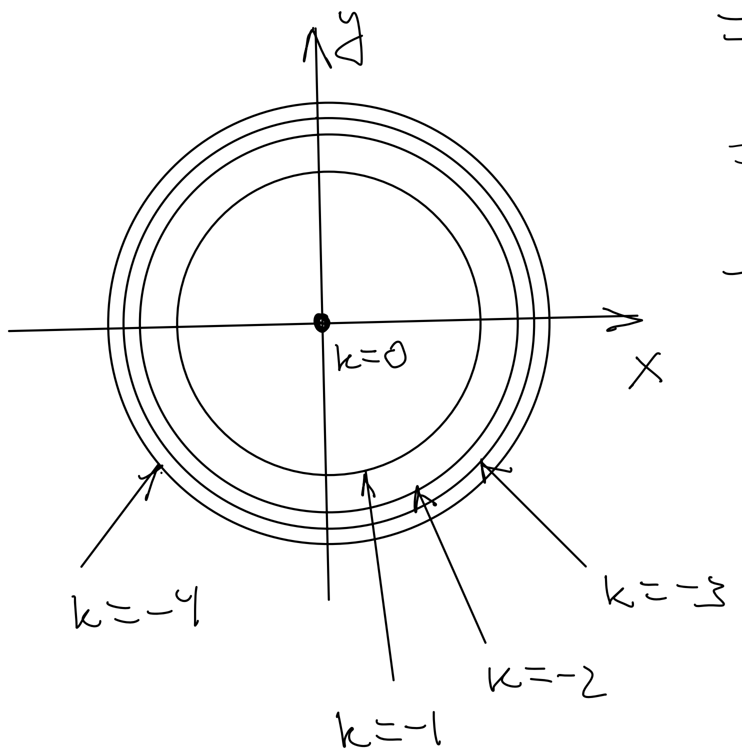
- $f(x,y) = \ln(1 - x^2 - y^2)$

$\ln(1-x^2-y^2) = k$

$\Rightarrow 1-x^2-y^2 = e^k$

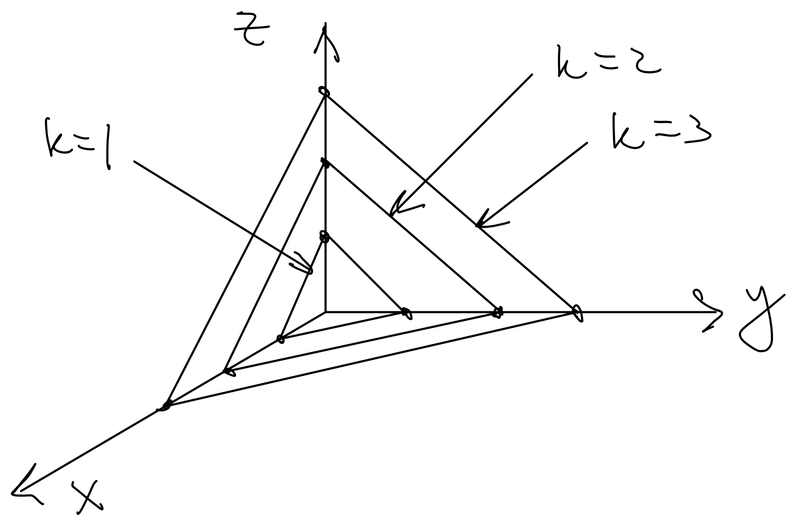
$\Rightarrow x^2+y^2 = 1-e^k > 0$ if $k<0$

– circle of the radius $\sqrt{1-e^k}$, centered at the origin.



- $g(x,y,z) = x + y + z$

$x+y+z=k$ – planes $\perp$ to $\langle 1,1,1\rangle$



Problem 13. Suppose that $f(x,y) = e^{-x}\sin y$. Show that the function f satisfies the differential equation $f_x = f_{yy}$.

$$f_x = -e^{-x}\sin y, \quad f_y = e^{-x}\cos y, \quad f_{yy} = -e^{-x}\sin y$$

$$\Rightarrow \quad f_x = -e^{-x}\sin y = f_{yy}$$

Problem 14. Find a given partial derivative.

- $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x,y) = \sqrt{x+y} + y$.

$$f_x = \frac{1}{2\sqrt{x+y}}; \quad f_y = \frac{1}{2\sqrt{x+y}} + 1$$

- $\frac{\partial^2 f}{\partial x \partial z}$ for $f(x,y,z) = x^{\ln yz}$.

$$f_z = \ln x \left(x^{\ln yz}\right) \frac{1}{\cancel{y}z}\cancel{y} = \frac{\ln x}{z} x^{\ln yz}$$

$$f_{zx} = \frac{1}{xz} x^{\ln yz} + \frac{\ln x}{z} x^{\ln yz - 1}$$

- $\frac{\partial u}{\partial z}$ for $z\sin(x+u) + u\cos(y-z) = 1$.

$$z\sin(x + u(x,y,z)) + u(x,y,z)\cos(y-z) = 1$$

$$\frac{\partial}{\partial z}\left(z\sin(x+u(x,y,z)) + u(x,y,z)\cos(y-z)\right) = \frac{\partial}{\partial z}(1) = 0$$

$$\sin(x+u(x,y,z)) + z\cos(x+u(x,y,z))\frac{\partial u}{\partial z} + \frac{\partial u}{\partial z}\cos(y-z)$$
$$- u(x,y,z)\sin(y-z) = 0$$

$$\sin(x+u) + z\cos(x+u)\frac{\partial u}{\partial z} + \frac{\partial u}{\partial z}\cos(y-z) - u\sin(y-z) = 0$$

$$\left(z\cos(x+u) + \cos(y-z)\right)\frac{\partial u}{\partial z} = u\sin(y-z) - \sin(x+u) \Rightarrow$$

$$\frac{\partial u}{\partial z} = \frac{u\sin(y-z) - \sin(x+u)}{z\cos(x+u) + \cos(y-z)}$$