Problem 8:

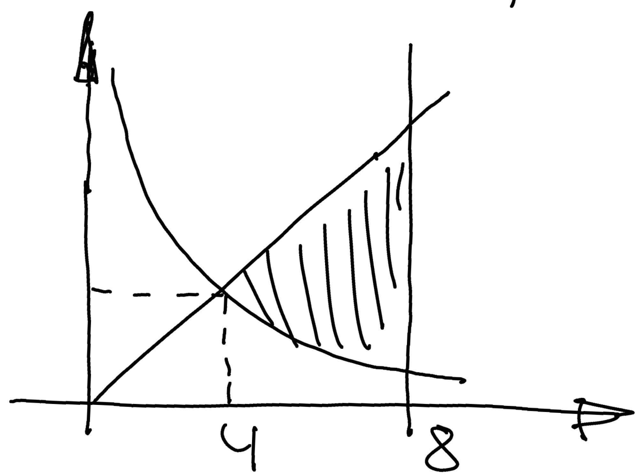
① $$\int_0^{\pi/2}\int_0^{\pi/3}(x\sin y - y\sin x)\,dy\,dx$$

$$= \int_0^{\pi/2}\left(\left(-x\cos y - \frac{y^2}{2}\sin x\right)\Big|_0^{\frac{\pi}{3}}\right)dx$$

$$= \int_0^{\frac{\pi}{2}}\left[-\frac{\pi^2}{18}\sin x + \frac{1}{2}x\right]dx$$

$$= \left(\frac{\pi^2}{18} \cos x + \frac{x^2}{4} \right) \Bigg|_0^{\pi/2} = \frac{\pi^2}{16} - \frac{\pi^2}{18}$$

2

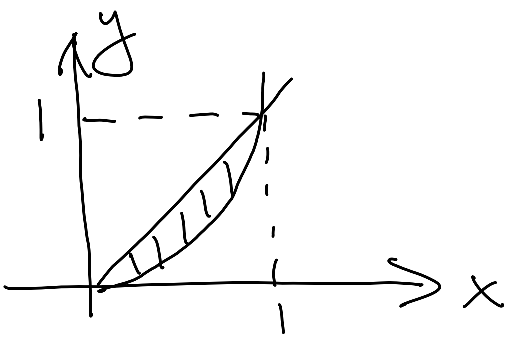


$$\int_4^8 \int_{16/x}^{x} x^2 \, dy \, dx = \int_4^8 \left(x^2 y \Big|_{\frac{16}{x}}^{x} \right) dx$$

$$= \int_4^8 (x^3 - 16x) \, dx = \left(\frac{x^4}{4} - 8x^2 \right) \Bigg|_4^8$$

$$= \frac{64^2}{4} - 8 \cdot 64 - 64 + 8 \cdot 16 = 576$$

3

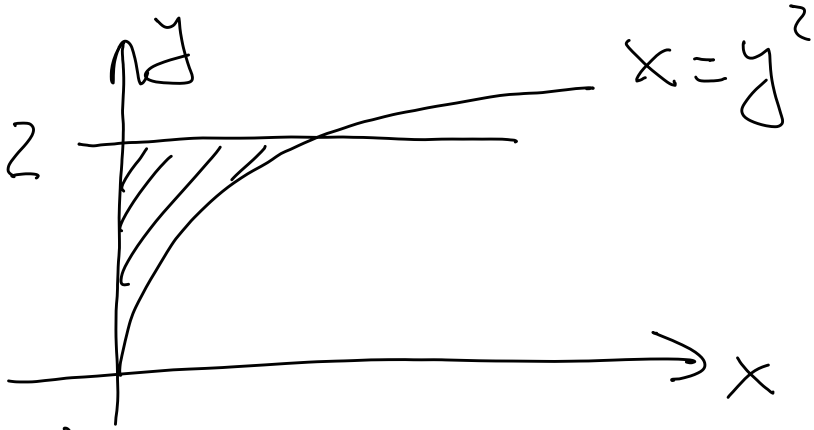


$$\int_0^1\int_{x^3}^{x}(x-1)\,dy\,dx$$

$$=\int_0^1 (x-1)(x-x^3)\,dx=\int_0^1 (x^2-x-x^4+x^3)\,dx$$

$$=\left(\frac{x^3}{3}-\frac{x^2}{2}-\frac{x^5}{5}+\frac{x^4}{4}\right)\Big|_0^1=-\frac{17}{60}$$

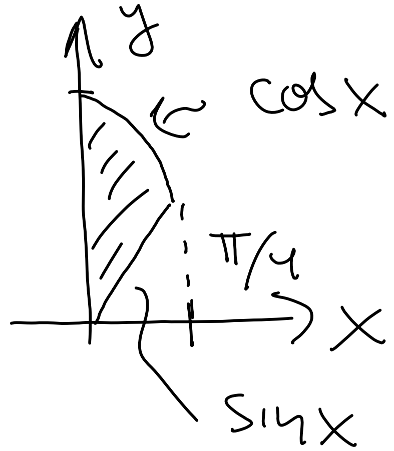
(4)



$$\int_0^2\int_0^{y^2}\sin y^3\,dx\,dy=\int_0^2 y^2\sin y^3\,dy\ \underset{dp=3y^2dy}{\overset{p=y^3}{=}}$$

$$=\frac{1}{3}\int_0^8 \sin p\,dp=\frac{1}{3}(1-\cos 8)$$

9.

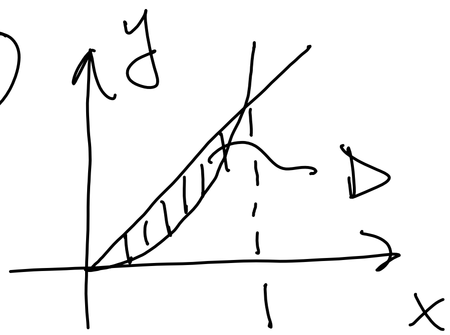


$$A = \int_0^{\pi/4}\int_{\sin x}^{\cos x} dy\,dx$$

$$= \int_0^{\pi/4} (\cos x - \sin x)\,dx$$

$$= (\cos x + \sin x)\Big|_0^{\frac{\pi}{4}} = \sqrt{2} - 1$$

10. ①



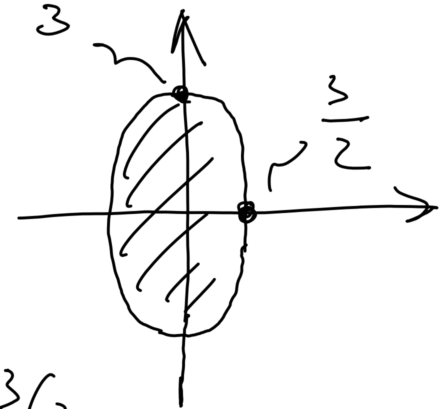
$$V = \iint_D (x^2 + 3y^2)\,dA$$

$$= \int_0^1\int_{x^2}^{x} (x^2+3y^2)\,dy\,dx = \int_0^1 \left[x^2 y + y^3\Big|_{x^2}^{x}\right] dx$$

$$= \int_0^1 (2x^3 - x^4 - x^6)\,dx = \frac{x^4}{2} - \frac{x^5}{5} - \frac{x^7}{7}$$

$$= \frac{1}{2} - \frac{1}{5} - \frac{1}{7}.$$

2



$$V = \int_{-3/2}^{3/2} \int_{-\sqrt{9-4x^2}}^{\sqrt{9-4x^2}} (y+3)\,dy\,dx$$

$$= \int_{-3/2}^{3/2} \left(\frac{y^2}{2} + 3y\right)\Bigg|_{-\sqrt{9-4x^2}}^{\sqrt{9-4x^2}} dx$$

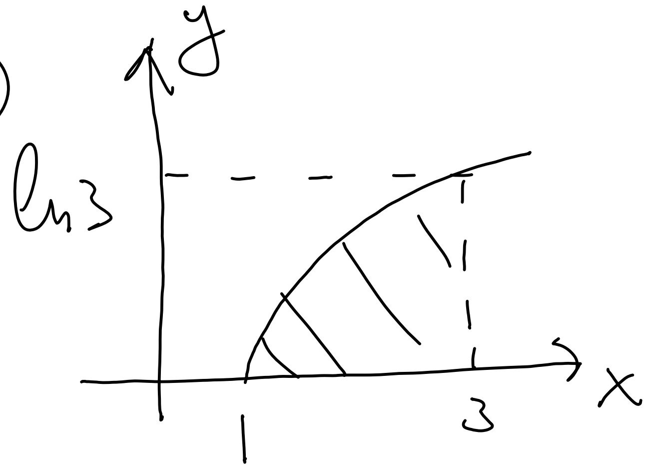
$$= \int_{-3/2}^{3/2} (9-4x^2)\,dx = \left(9x - \frac{4x^3}{3}\right)\Bigg|_{-\frac{3}{2}}^{\frac{3}{2}}$$

$$= 18.$$

Problem 11. 1 $= \int_0^2 \int_0^{x^2} e^{x^3}\,dy\,dx$

$$= \int_0^2 x^2 e^{x^3}\,dx \underset{du = 3x^2dx}{\overset{u = x^3}{=}}$$

$$= \frac{1}{3}\int_0^8 e^u\,du = \frac{1}{3}\left(e^8-1\right)$$

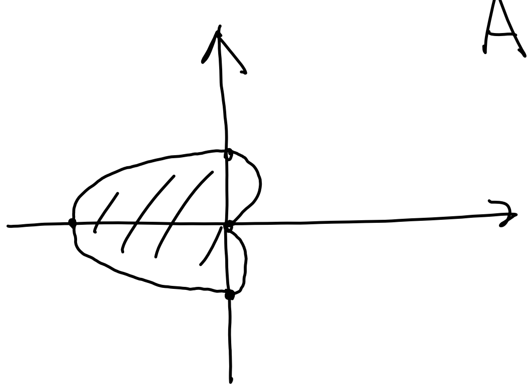
② 

$$= \int_0^{\ln 3}\int_{e^y}^{3} x\,dx\,dy$$

$$= \frac{1}{2}\int_0^{\ln 3}\left(9-e^{2y}\right)dy$$

$$= \frac{1}{2}\left(9y-\frac{1}{2}e^{2y}\right)\Big|_0^{\ln 3} = \frac{1}{2}\left(9\ln 3 - \frac{9}{2}+\frac{1}{2}\right) = \frac{1}{2}\left(9\ln 3-4\right)$$

Problem 12.1:

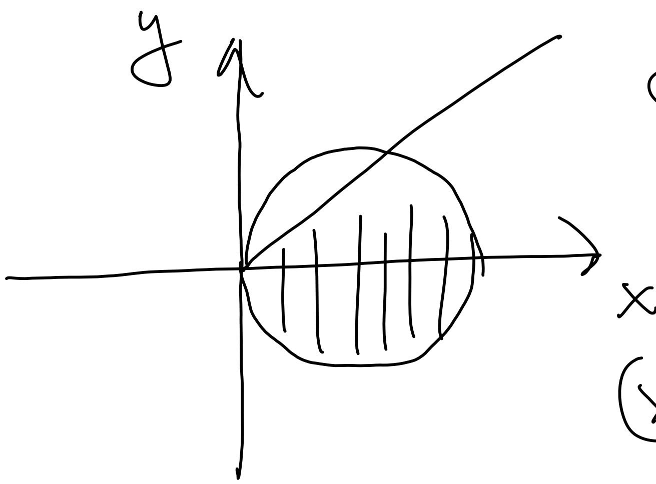


$$A = \int_0^{2\pi}\int_0^{1-\cos\theta} r\,dr\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}\left(r^2\Big|_0^{1-\cos\theta}\right)d\theta = \frac{1}{2}\int_0^{2\pi}(1-\cos\theta)^2\,d\theta$$

$$= \frac{1}{2}\int_0^{2\pi}(1-2\cos\theta+\cos^2\theta)\,d\theta = \frac{1}{2}\int_0^{2\pi}\Big(1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta)\Big) = \frac{3\pi}{2}.$$

Problem 13.2



$y=x$: $\cancel{r}\sin\theta = \cancel{r}\cos\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$(x-1)^2+y^2=1$:

$x^2-2x+\cancel{1}+y^2=\cancel{1}$

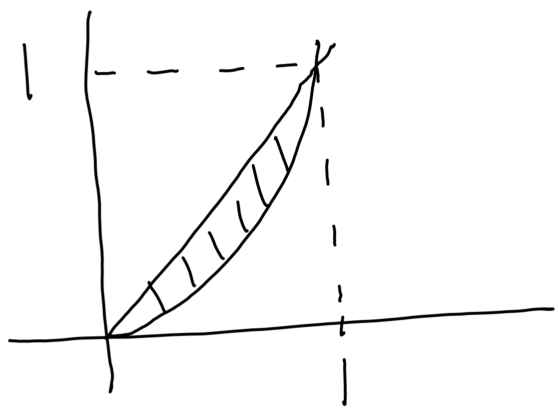
$\to r^2-2r\cos\theta=0 \to r=2\cos\theta$

$$A=\int_{-\pi/2}^{\pi/4}\int_0^{2\cos\theta} r\,dr\,d\theta = 2\int_{-\pi/2}^{\pi/4}\cos^2\theta\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/4} (1+\cos 2\theta)\, d\theta = \left(\theta + \frac{1}{2}\sin 2\theta\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{4}}$$

$$= \frac{3\pi}{4} + \frac{1}{2}$$

Problem 14.2



$y = x \iff \theta = \frac{\pi}{4}$

$y = x^2$:

$r\sin\theta = r^2\cos^2\theta$

$\Rightarrow r = \dfrac{\sin\theta}{\cos^2\theta}$

$$\int_0^{\pi/4}\int_0^{\frac{\sin\theta}{\cos^2\theta}} r\cdot r\, dr\, d\theta = \int_0^{\pi/4} \frac{r^3}{3}\Big|_0^{\frac{\sin\theta}{\cos^2\theta}} d\theta$$

$$= \frac{1}{3}\int_0^{\pi/4} \frac{\sin^3\theta}{\cos^6\theta}\, d\theta \underset{du = -\sin\theta\, d\theta}{\overset{u=\cos\theta}{=}} -\frac{1}{3}\int_1^{\sqrt{2}/2} \frac{1-u^2}{u^6}\, du$$

$$= \frac{1}{3}\int_{\sqrt{2}/2}^{1} (u^{-6} - u^{-4})\, du = \frac{1}{3}\left(-\frac{u^{-5}}{5} + \frac{u^{-3}}{3}\right)\Bigg|_{\frac{\sqrt{2}}{2}}^{1}$$

$$= \frac{1}{3}\left(\frac{1}{3} - \frac{1}{5}\right) - \frac{1}{3}\left(-\frac{1}{5}(\sqrt{2})^{5} + \frac{1}{3}(\sqrt{2})^{3}\right)$$