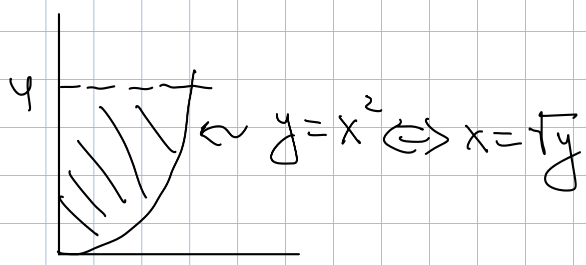
(8.3) $\iint_D x(1+y^2)^{-1/2}\,dA = \int_0^4\int_0^{\sqrt{y}} x(1+y^2)^{-1/2}\,dx\,dy = \int_0^4 (1+y^2)^{-1/2}\left(\frac{x^2}{2}\Big|_0^{\sqrt{y}}\right)dy$

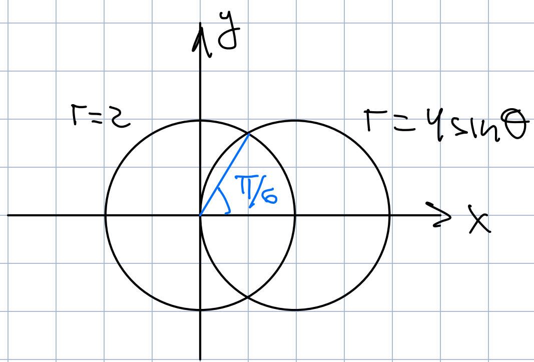


$$=\frac{1}{2}\int_0^4 (1+y^2)^{-1/2}\,y\,dy \;\underset{du=2y\,dy}{\overset{u=y^2+1}{=}}$$

$$=\frac{1}{4}\int_1^{17} u^{-1/2}\,du = \frac{1}{2}u^{1/2}\Big|_1^{17} = \frac{1}{2}\left(\sqrt{17}-1\right)$$

(12.2) $r=4\sin\theta \to r^2=4r\sin\theta \to x^2+y^2=4y \to$

$x^2+y^2-4y+4=4 \to x^2+(y-2)^2=4$



Intersect when $4\sin\theta=2 \Rightarrow$

$\sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$

$$\text{Area} = 2(\text{Area above x-axis}) = 2\int_0^{\pi/6}\int_0^{2} r\,dr\,d\theta + 2\int_{\pi/6}^{\pi/2}\int_0^{4\sin\theta} r\,dr\,d\theta$$

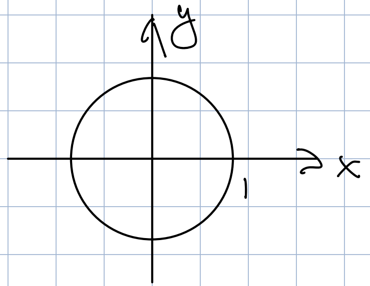
$$=2\int_0^{\pi/6}d\theta\int_0^2 r\,dr + \int_{\pi/6}^{\pi/2}\left(r^2\Big|_0^{4\sin\theta}\right)d\theta = \frac{2\pi}{3} + \int_{\pi/6}^{\pi/2} 16\sin^2\theta\,d\theta$$

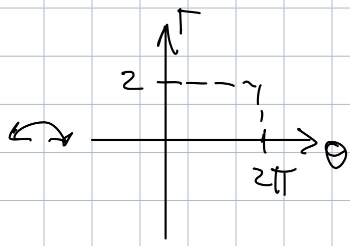
$$=\frac{2\pi}{3}+8\int_{\pi/6}^{\pi/2}(1-\cos 2\theta)\,d\theta = \frac{2\pi}{3}+8\left(\theta-\frac{1}{2}\sin 2\theta\right)\Big|_{\pi/6}^{\pi/2}$$

$$=\frac{2\pi}{3}+8\left(\frac{\pi}{2}-\frac{\pi}{6}+\frac{1}{2}\sin\frac{\pi}{3}\right) = \frac{10\pi}{3}+4\sin\frac{\pi}{3}$$

13.1

$$\iint_R e^{-x^2-y^2}\,dA = \int_0^{2\pi}\int_0^2 e^{-r^2}\, r\,dr\,d\theta \overset{u=-r^2}{\underset{du=-2r\,dr}{=}} \frac{1}{2}\int_0^{2\pi}\int_{-4}^{0} e^u\,du\,d\theta$$





$$= 2\pi\cdot\frac{1}{2}e^u\Big|_{-4}^{0} = \pi\left(1-e^{-4}\right)$$

14.1

$$\int_0^1\int_0^{\sqrt{1-y^2}} \cos(x^2+y^2)\,dx\,dy = \int_0^{\pi/2}\int_0^1 \cos r^2\, r\,dr\,d\theta$$

$$\overset{u=r^2}{=} \frac{1}{2}\int_0^{\pi/2}\int_0^1 \cos u\,du = \frac{\pi}{4}\sin u\Big|_0^1 = \frac{\pi}{4}\sin 1.$$